PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

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Phil. Trans. R. Soc. Lond. A 1995 **352**, 443-452
doi: 10.1098/rsta.1995.0081

Distortion of the stagnation-point flow due to cross-stream vorticity in the external flow

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The structure of the stagnation-point flow in the presence of weak steady cross-stream vorticity in the external flow is investigated. A specific case of the two-dimensional basic forward stagnation-point flow past a circular cylinder is considered with the external three-dimensional vortical disturbance taken to be periodic in the spanwise direction with a wavelength $\lambda^* \leqslant \lambda_N^* = \pi D/(Re_D)^{1/2}$, where D is the diameter of the cylinder and Re_D is the flow Reynolds number. It is shown that the presence of weak but finite streamwise vorticity, with $\lambda^* \leqslant \lambda_N^*$ in the external flow, can be supported by the flow in the stagnation zone, leading to a substructure of counter-rotating streamwise eddies in the boundary layer. The magnitude of the streamwise vorticity in the boundary layer is found to match with that in the external flow for $\lambda^* \leqslant \lambda_N^*$; it is of much smaller order for $\lambda^* > \lambda_N^*$, which corresponds to a disturbance of the type considered by Hämmerlin (1955).

1. Introduction

The importance of the role played by the presence of upstream grid turbulence in transition experiments involving flow over bluff bodies or on a flat plate with rounded leading edge is well recognized. Although low turbulence levels may be achieved in the upstream flow through careful flow management, there is a concern that *any* vorticity in the free stream may be sufficiently amplified in the vicinity of the forward stagnation zone of the body to bear a greater degree of free-stream influence on the process of transition than anticipated. It is therefore of interest to examine the flow structure in the stagnation zone in the presence of weak vorticity in the free stream. In an experimental study of amplification of streamwise vorticity by stagnation-point flow, Sadeh *et al.* (1980, 1970) considered impingement of cross-stream vorticity on to a cylinder which is aligned with its axis normal to both the streamwise direction and to the direction of the oncoming vorticity. The configuration considered was similar to that depicted in figure 1. They performed experiments with spanwise wavelengths in the range 4–40 mm, identifying the existence of scales which are amplified by the flow in the stagnation zone and observing an associated persistent substructure consisting of a coherent array of eddies in the stagnation zone. In particular, they delineated a spanwise scale $\lambda_N^* = \pi D/Re_D^{1/2}$ (= 5.6 mm in their case), where D is the diameter of the cylinder and Re_D is the flow Reynolds number, such that the dynamics of scales larger than λ_N^* are dominantly governed by inviscid dynamics involving vortex stretching and vorticity amplification, while

Phil. Trans. R. Soc. Lond. A (1995) **352**, 443–452
Printed in Great Britain

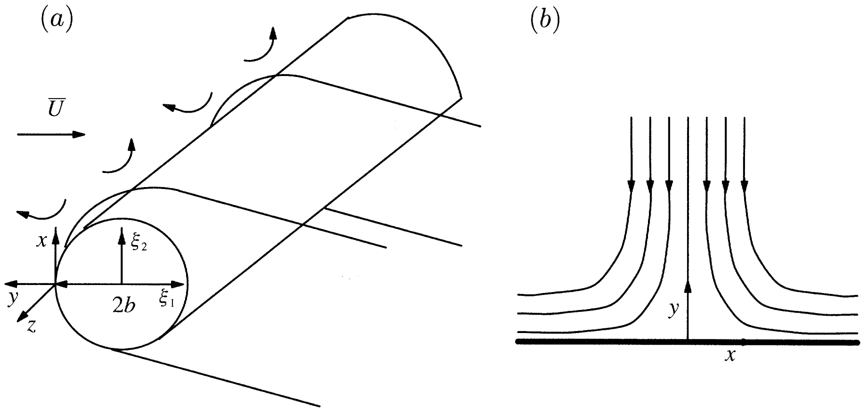


Figure 1. (a) Flow past a circular cylinder, and (b) stagnation-point flow.

scales smaller than λ_N^* are governed by viscous effects associated with the flow in the stagnation region. This is consistent with other experimental investigations, all of which report the appearance, in the two-dimensional stagnation flow region, of streamwise vortices of the scale of the boundary-layer thickness. Further relevant experimental and theoretical papers include those of Sutera *et al.* (1963), Sutera (1965), Kestin & Wood (1970) and Bearman (1972).

In this paper, the structure of the flow in the stagnation zone of a bluff body in the presence of weak steady streamwise vorticity is examined. The vortical flow corresponds to a Görtler-type disturbance (Görtler 1955) to the stagnation zone flow, varying in the spanwise direction on a length scale $\lambda^* \leqslant \lambda_N^*$. This is in contrast to the work of Goldstein *et al.* (1992) who considered the case of external vortical disturbance to flow over a flat plate of rounded leading edge; in their case $\lambda^* = O(D)$ so that the dynamics of vorticity are governed by inviscid flow and they were able to treat the stagnation region using rapid-distortion theory. On the other hand, the influence of viscosity on turbulent flow near a stagnation-point flow was considered by Goldstein (1984), although the details and the approach of that paper are rather different from those described here.

Görtler-type disturbances to the stagnation flow region have received considerable attention in the literature (Hämmerlin 1955; Wilson & Gladwell 1978; Brattkus & Davis 1991) in the case where the disturbances originate in the boundary layer and decay exponentially away from the rigid wall. These occur for a wavelength $\lambda^* > \lambda_N^*$, so that the wavenumber is less than 1 in Hämmerlin's notation. However, there exist eigensolutions to the governing disturbance vorticity equations, corresponding to other spanwise wavelengths, $\lambda^* < \lambda_N^*$ (eigenvalues), which grow algebraically away from the wall and it is shown here that these match with the external flow involving weak steady cross-stream vorticity. This leads to the suggestion that the flow in the stagnation zone will support the presence in the external stream of weak, but amplified, cross-stream vorticity of spanwise wavelengths $\lambda^* < \lambda_N^*$, consistent with experimental observation; the corresponding boundary-layer flow consists of a substructure of counter-rotating streamwise eddies. The relationship between the free-stream vorticity and the boundary-layer vorticity is determined and a matching is achieved. The solutions obtained provide useful upstream boundary conditions for numerical simulations which involve following development of streamwise vorticity in the boundary layer on a blunt body or a flat plate with rounded leading edge in the presence of disturbances in the external flow. It is believed that the solutions presented here are consistent with the experimental observations of Sadeh & Brauer (1980) at wavelengths below and above λ_N^*.

Specifically, the stagnation-point flow of an incompressible fluid of uniform density, modified by the presence of small streamwise vorticity $O(\epsilon)$ in the external flow, is considered. Cartesian coordinates (x^*, y^*, z^*), as shown in figure 1*b*, are defined, with the stagnation line along $x^* = y^* = 0$ and position, time, fluid velocity and vorticity are non-dimensionalized with respect to typical length (l; $l = b$ in figure 1*a*) and velocity ($\overline{U}$) scales associated with the external flow as $\underline{x}^* = l\underline{x}$, $t^* = l\overline{U}^{-1}t$, $\underline{u}^* = \overline{U}\underline{u}(x, y, z)$, $\underline{\omega}^* = \overline{U}l^{-1}\underline{\omega}$. The flow Reynolds number, considered to be large, is given by $Re = \overline{U}l/\nu$, where ν is the kinematic viscosity. The streamwise vorticity is periodic in the spanwise direction, with a characteristic wavelength $2\pi(2Re)^{-1/2}/\beta$ for some β. β may be considered to be an eigenvalue for neutral disturbances to the basic flow. In view of this scaling of the spanwise variation of flow, it is convenient to introduce a stretched variable:

$$Z = (2Re)^{1/2}z. \tag{1.1}$$

The external flow in the vicinity of the stagnation flow is obtained in § 2 and is matched to the inner viscous-flow solution obtained in § 3 to determine the flow structure. A Reynolds number may seem anomalous here, but it is necessary for any extensions along the body beyond the stagnation region; for a discussion see Stuart (1963).

2. External flow

For definiteness, the external flow past a cylinder of unit radius is considered, although the analysis can be extended to consider the forward stagnation region of any two-dimensional body including a flat plate with rounded leading edge. It is convenient first to express the external problem in terms of the more usual coordinate system (ξ_1, ξ_2, z), shown in figure 1*a* with the origin at the centre of the cylinder. Thus, $\xi_1 = -1 - y$ and $\xi_2 = x$. The external velocity $\underline{U}$ is taken to be of the form

$$\underline{U} = (U_{01}(\xi_1, \xi_2), U_{02}(\xi_1, \xi_2), 0) + \epsilon\underline{U}_1(\xi_1, \xi_2, Z) + O(\epsilon^2)$$

with
$$U_{01} = \frac{\partial \Psi_0}{\partial \xi_2}, \quad U_{02} = -\frac{\partial \Psi_0}{\partial \xi_1}, \quad \Psi_0 = \xi_2(1 - ({\xi_1}^2 + {\xi_2}^2)^{-1}). \tag{2.1}$$

$\Psi_0(\xi_1, \xi_2)$ is the stream function for the basic flow, which is irrotational. The $O(\epsilon)$ term in (2.1) represents the vortical contribution together with a potential flow required to satisfy the boundary condition at the surface of the cylinder. The vorticity in the external region is given by $\underline{\Omega} = \epsilon\underline{\Omega}_1 + O(\epsilon^2)$, where, in the (ξ_1, ξ_2, z) frame,

$$\underline{\Omega}_1 = \nabla \times \underline{U}_1 = (\Omega_{11}(\xi_1, \xi_2)\sin(\beta Z), \Omega_{12}(\xi_1, \xi_2)\sin(\beta Z), \Omega_{13}(\xi_1, \xi_2)\cos(\beta Z)). \tag{2.2}$$

If we substitute for $\underline{\Omega}$ in the vorticity equation, then to leading order in ϵ and Re, we have

$$\left.\begin{aligned} U_{01}\frac{\partial \Omega_{11}}{\partial \xi_1} + U_{02}\frac{\partial \Omega_{11}}{\partial \xi_2} &= \Omega_{11}\frac{\partial U_{01}}{\partial \xi_1} + \Omega_{12}\frac{\partial U_{01}}{\partial \xi_2} - 2\beta^2\Omega_{11}, \\ U_{01}\frac{\partial \Omega_{12}}{\partial \xi_1} + U_{02}\frac{\partial \Omega_{12}}{\partial \xi_2} &= \Omega_{11}\frac{\partial U_{02}}{\partial \xi_1} + \Omega_{12}\frac{\partial U_{02}}{\partial \xi_2} - 2\beta^2\Omega_{12}, \\ U_{01}\frac{\partial \Omega_{13}}{\partial \xi_1} + U_{02}\frac{\partial \Omega_{13}}{\partial \xi_2} &= -2\beta^2\Omega_{13}. \end{aligned}\right\} \tag{2.3}$$

We consider the case where

$$\Omega_{13} = O(Re^{-1/2}|\underline{\Omega}|), \tag{2.4}$$

which is consistent with an appropriate initial condition upstream. Thus, to leading order,

$$\Omega_{13}\cos(\beta Z) = \frac{\partial U_{12}}{\partial \xi_1} - \frac{\partial U_{11}}{\partial \xi_2} \equiv 0, \quad \text{and} \quad U_{11} = \frac{\partial \Phi_1}{\partial \xi_1}, \quad U_{12} = \frac{\partial \Phi_1}{\partial \xi_2}$$

for some potential function Φ_1. Then

$$\Omega_{11} = \frac{\partial \chi}{\partial \xi_2}, \quad \Omega_{12} = -\frac{\partial \chi}{\partial \xi_1}, \tag{2.5}$$

where $\chi(\xi_1,\xi_2)\sin(\beta Z) = U_{13} - (2Re)^{1/2}(\partial \Phi_1/\partial Z)$.

On substituting (2.5) in the first or second of equations (2.3), integrating with respect to ξ_2 or ξ_1, respectively, and applying conditions for $\xi_1 \to -\infty$, we obtain

$$U_{01}\frac{\partial \chi}{\partial \xi_1} + U_{02}\frac{\partial \chi}{\partial \xi_2} = -2\beta^2\chi. \tag{2.6}$$

A solution to this linear equation for χ may be found by the method of characteristics. The characteristics are given by

$$\Psi_0(\xi_1,\xi_2) = c,$$

for constant values of c, and by

$$\int^{\xi_1} \frac{-2\beta^2\,\mathrm{d}\xi_1}{U_{01}(\xi_1,\xi_2(\xi_1,c))} = \ln\chi + \text{const.}$$

Thus, the solution of (2.6) is

$$\chi = \overline{\chi}\exp\left[-2\beta^2\int^{\xi_1}\frac{\mathrm{d}\xi_1}{U_{01}(\xi_1,\xi_2(\xi_1,c))}\right], \tag{2.7}$$

where $\overline{\chi}$ is a function of Ψ_0, and c is replaced by Ψ_0 after the integration; on matching with the viscous solution obtained in § 3, we find that $\overline{\chi}$ is a constant.

In the vicinity of the forward stagnation line, $\xi_1 = -1$, $\xi_2 = 0$, we have in terms of x $(= \xi_2)$ and y $(= -1 - \xi_1)$ variables, for $|y|$, $|x| \ll 1$,

$$\Psi_0 \simeq x(2y + x^2 - 3y^2 + O(x^2y)),$$

so that, from (2.1),

$$U_{01}{}^{-1} \simeq \frac{1}{2y} + \frac{3}{4} - \frac{3x^2}{4y^2} + O\left(y, \frac{x^2}{y}, \frac{x^4}{y^3}\right).$$

Here we limit consideration to the leading-order stagnation-point flow, approximating U_{01}^{-1} by the leading term in the expansion, so that, from (2.7),

$$\chi \simeq \overline{\chi}y^{\beta^2},$$

and match this with an appropriate leading-order solution for the inner problem in § 3. Thus, in view of (2.2) and (2.5) and on using $y \to y_a = y - \delta$, where $\delta = 0.6479(2Re)^{-1/2}$ is the displacement thickness, we have that the vorticity in the external flow in the vicinity of the stagnation point is given, in the (x, y, z) frame (with Ω_{12} corresponding to the x-component of vorticity), by

$$\underline{\Omega}_1 \sim (\beta^2 \overline{\chi} y_a^{\beta^2-1} \sin(\beta Z), 0, 0). \tag{2.8}$$

A higher-order solution to $\underline{\Omega}_1$ may be obtained from (2.7) by considering more terms in the expansion for U_{01}^{-1} and matching the resulting expression for the vorticity with a corresponding higher-order solution to the inner problem.

3. Viscous stagnation region

In addition to the variable Z defined in (1), it is convenient to define a stretched variable η, for consideration of the flow in the viscous stagnation region, as

$$\eta = (2Re)^{1/2} y \tag{3.1}$$

and take the velocity field in the region to be of the form involving a basic stagnation flow subject to a perturbation:

$$\left.\begin{aligned} \underline{u} &= \underline{u}_0 + \epsilon \underline{u}_1 + O(\epsilon^2), \\ \underline{u}_0 &= (2xu_0(\eta), 2(2Re)^{-1/2} v_0(\eta), 0), \\ \underline{u}_1 &= (xu_1(\eta)\cos(\beta Z), (2Re)^{-1/2} v_1(\eta)\cos(\beta Z), (2Re)^{-1/2} w_1(\eta)\sin(\beta Z)). \end{aligned}\right\} \tag{3.2}$$

Then the vorticity $\underline{\omega}$ is given by

$$\left.\begin{aligned} \underline{\omega} &= (0, 0, \omega_{03}) + \epsilon \underline{\omega}_1, \quad \omega_{03} = -2x(2Re)^{1/2} u_0', \\ \underline{\omega}_1 &= (\zeta_1 \sin(\beta Z), -(2Re)^{1/2} \beta x u_1 \sin(\beta Z), -x(2Re)^{1/2} u_1{}' \cos(\beta Z)), \\ \zeta_1 &= w_1{}' + \beta v_1, \end{aligned}\right\} \tag{3.3}$$

where a prime denotes differentiation with respect to η. On substituting these expressions into the equations of motion and, respectively, equating zeroth- and first-order terms in ϵ to zero, we obtain

$$\left.\begin{aligned} \nabla \cdot \underline{u}_0 = 0, \quad \nabla \cdot \underline{u}_1 = 0, \quad \underline{u}_0 \cdot \nabla \omega_{03} = Re^{-1} \nabla^2 \omega_{03}, \\ \underline{u}_0 \cdot \nabla \underline{\omega}_1 + \underline{u}_1 \cdot \nabla \underline{\omega}_0 = \underline{\omega}_0 \cdot \nabla \underline{u}_1 + \underline{\omega}_1 \cdot \nabla \underline{u}_0 + Re^{-1} \nabla^2 \underline{\omega}_1, \end{aligned}\right\} \tag{3.4}$$

subject to the boundary conditions at the rigid surface and in the external stream. Upon substituting for ω_{03}, $\underline{u}_0$, $\underline{u}_1$ and $\underline{\omega}_1$ from (3.2–3.3) into (3.4), we, respectively, obtain

$$\left.\begin{aligned} u_0 + v_0' = 0, \quad u_1 + v_1' + \beta w_1 = 0, \\ (u_0{}^2)' + (v_0 u_o{}')' - u_0{}''' = 0, \\ \zeta_1{}'' - v_0 \zeta_1{}' - (v_0{}' + \beta^2)\zeta_1 = 0, \\ u_1{}'' - v_0 u_1{}' - (2u_0 + \beta^2) u_1 = u_0{}' v_1, \end{aligned}\right\} \tag{3.5}$$

with the corresponding equation associated with the spanwise component of $\underline{\omega_1}$ given by the derivative of the last of (3.5).

Table 1. *Asymptotic behaviour of zeta and v for different values of n*

n	ζ_1	v_1
0	c_0	$\sim d_0$
1	$c_1\eta_a$	$d_1\eta_a$
2	$c_2({\eta_a}^2+1)$	$d_2({\eta_a}^2+\frac{5}{3})$
3	$c_3({\eta_a}^3+3\eta_a)$	$d_3({\eta_a}^3+\frac{9}{2}\eta_a)$
4	$c_4({\eta_a}^4+2{\eta_a}^2+3)$	$d_4({\eta_a}^4+\cdots)$
n	$c_n({\eta_a}^n+\frac{1}{2}n(n-1){\eta_a}^{n-2}+\cdots)$	$d_n({\eta_a}^n+\cdots)$

(*a*) *Outer solution*

In the outer region $\eta \gg 1$, we have

$$u_0 \to 1, \quad v_0 \to -\eta_a, \tag{3.6}$$

where $\eta_a = \eta - 0.6479$. Thus, in view of (3.5), ζ_1 and u_1 satisfy

$$\zeta_1'' + \eta_a\zeta_1' - n\zeta_1 = 0, \tag{3.7}$$

$$u_1'' + \eta_a u_1' - (n+3)u_1 = 0, \tag{3.8}$$

where $n = \beta^2 - 1$. Furthermore, from (3.3) and (3.5), we have that

$$v_1'' - \beta^2 v_1 = -\beta\zeta_1 - u_1', \quad w_1 = -(u_1 + v_1')/\beta. \tag{3.9}$$

Equations (3.7) and (3.8) correspond to Hermite's equation and have an exponentially decaying solution and an algebraic solution. The latter grows in η_a if $n \geqslant 0$ (and is a polynomial if n is an integer), but decays if $n < 0$. In order to illustrate the nature of the solutions to these equations, we consider in detail the cases $n = 0, 1, 2$, corresponding to spanwise wavenumbers $\beta^2 = 1, 2, 3$; this choice implies that ζ_1 has a polynomial structure at infinity. Non-integer values may equally be considered without difficulty and would imply a solution with an algebraic structure at infinity. In general, the differential system constitutes an eigenvalue problem for n, albeit with a continuous spectrum and an unusual boundary condition that the solution has an algebraic or, in special cases, a polynomial structure at infinity. We further require that, to the order of the stagnation-point flow, $u_1 \to 0$ exponentially as $\eta \to \infty$. The latter requirement is dictated by the matching procedure. Thus we obtain the results displayed in table 1.

Here $d_n = c_n/\surd(n+1)$ and c_n are constants to be determined by matching. If we express the limiting solution for $\underline{\omega}_1$, given by (3.3), with ζ_1 and u_1 given in table 1, in terms of y_a and compare it with the external limiting expression for $\underline{\Omega}_1$ given by (2.8), we obtain that

$$c_n = (n+1)\overline{\chi}(2Re)^{-n/2}. \tag{3.10}$$

The corresponding asymptotic expressions for w_1 are obtained using $w_1 \sim -v_1'/\beta$. With the choice (3.10), the streamwise vorticity in the boundary layer matches that in the external flow. Matching requires that $\overline{\chi}$ be a constant, although this aspect could change if the solution were continued along the body beyond the stagnation region. Brattkus & Davis (1991) discuss more general disturbances, the x-wise behaviour of which can involve a higher power of x than the linear dependence considered in (3.2).

Table 2. *The values of* ζ_1, ζ_1' *and* u_1' *at the wall for different values of* n

n	$\zeta_1(0)/c_n$	$\zeta_1'(0)/c_n$	$u_1'(0)/c_n$
0	−0.794	1.19	−0.032
1	−0.803	1.53	−0.035
2	−1.074	2.39	−0.047

(*b*) *Inner solution*

If we integrate the equation for u_0 in (3.5) with respect to η and use $u_0 \to 1$ as $\eta \to \infty$, we obtain

$$u_0 = f'(\eta), \quad v_0 = -f(\eta), \tag{3.11}$$

where

$$f''' + ff'' - f^{2\prime} + 1 = 0. \tag{3.12}$$

The equation for streamwise vorticity ζ_1 in (3.5) becomes

$$\zeta_1'' + f\zeta_1' - (\beta^2 - f')\zeta_1 = 0 \tag{3.13}$$

and from the first of the equations for u_1 in (3.5) we have

$$u_1'' + fu_1' - (2f' + \beta^2)u_1 = f''v_1. \tag{3.14}$$

The boundary conditions to be satisfied at the rigid surface are

$$f = f' = u_1 = v_1 = w_1 = v_1' = 0 \text{ at } \eta = 0. \tag{3.15}$$

As $\eta \to \infty$, ζ, v_1 and $w_1 \to$ a polynomial structure of the form given in table 1, while $f' \to 1$ and $u_1 \to 0$. We evaluate f, u_1 and ζ_1 in the inner layer subject to these outer boundary conditions.

Equations (3.12)–(3.14) were integrated using a two-point boundary-value shooting method and fourth-order Runge–Kutta integration. Integration in η was carried out from $\eta = 0$ to $\eta = \eta_\infty$, where the value of $\eta_\infty = 6.19$ was found to be adequate; the value $f''(0) = 1.232\,588$ was used in the computation. The values of ζ_1, ζ_1' and u_1' at the wall for different values of n are given in table 2.

The profiles for the velocity contributions f', u_1, v_1 and w_1, and for the streamwise vorticity ζ_1 as a function of η, are shown in figure 2 for three values of β^2 $(= n+1)$; leading-order solutions in the inner layer are shown for f' and u_1. The appropriate polynomial behaviour for large η, as described in table 1, is apparent from the figures. It is interesting that ζ_1 near $\eta = 0$ has opposite sign to that in the outer flow. This is illustrated further in figure 3, which shows the disturbance streamwise vorticity contours, given by $\zeta_1(\eta)\sin\beta Z$, for the particular case of $\beta^2 = 2$; other cases have similar patterns. A plot of the 'streamlines' for the disturbance velocity $(v_1\cos\beta Z, w_1\sin\beta Z)$ in the $Z-\eta$ plane for the case $\beta^2 = 2$ is shown in figure 4*a* and the corresponding projected 'streamlines' in the $Z-\eta$ plane for $\epsilon = 0.2$ are shown in figure 4*b*.

4. Conclusion

In this paper, the structure of the stagnation zone flow in the presence of weak steady cross-stream vorticity in the external stream has been considered for the case of a spanwise scale of flow variation $O(Re^{-1/2})$. The solution for the specific case

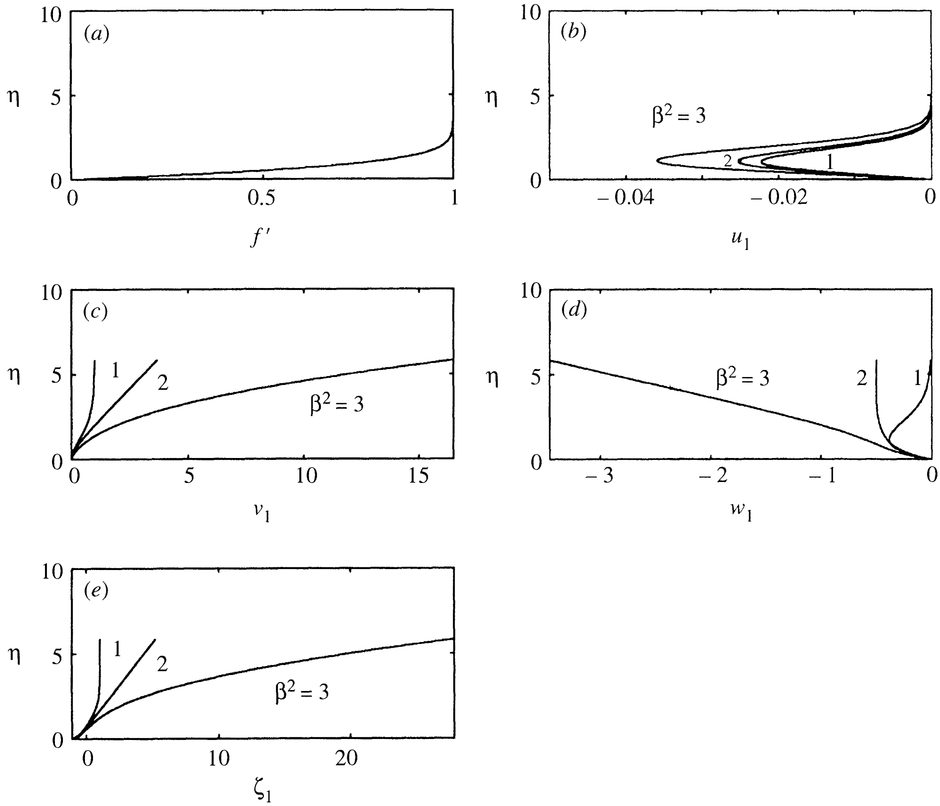


Figure 2. Basic flow function f' (equation 3.11), and perturbation velocity components (*b*) u_1, (*c*) v_1, (*d*) w_1 and (*e*)streamwise vorticity ζ_1 as a function of η for three values of spanwise wavenumber β.

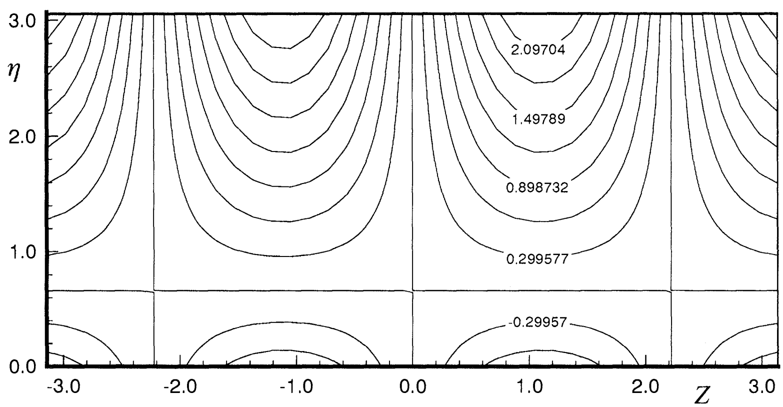
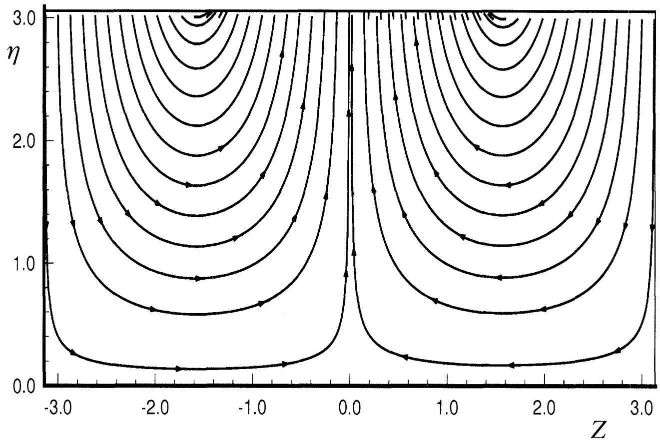


Figure 3. Streamwise disturbance vorticity contours in the η–Z plane.

of flow past a cylinder suggests that the stagnation-point flow will support, in the external flow, the presence of weak, but amplified, steady cross-stream vorticity with $\lambda^* \leqslant \lambda_N^* (= \pi D/{Re_D}^{1/2})$, leading to the existence of a persistent substructure consisting of an array of coherent counter-rotating streamwise eddies in the stagnation



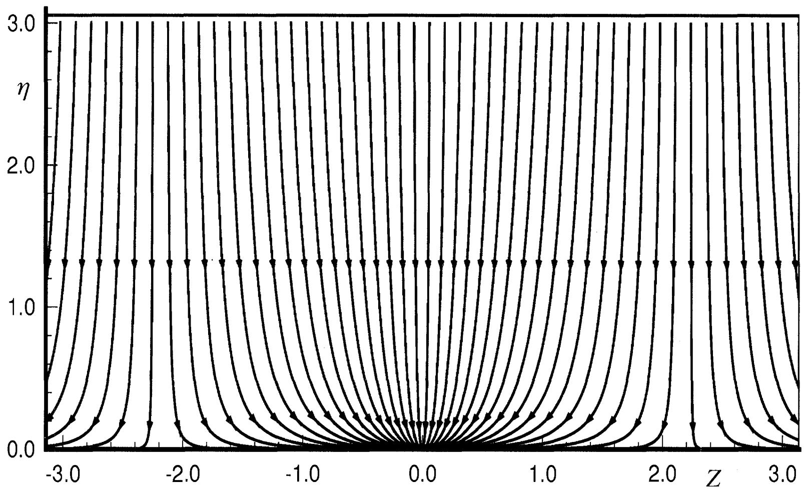


Figure 4. (*a*) Disturbance 'streamlines' in the η–Z plane and (*b*) 'streamlines' in the η–Z plane for $\epsilon = 0.2$.

region; this is consistent with experimental observation. For $\lambda^* \leqslant \lambda_N^*$, the magnitude of the streamwise vorticity in the boundary layer matches with that in the external flow; for $\lambda^* > \lambda_N^*$, which corresponds to a disturbance of the type considered by Hämmerlin (1955), it is of much smaller order.

The results presented here can be related to previous work. As summarized by Stuart (1963), the existence of velocity fluctuations in the neighbourhood of the stagnation point of a body has been known since 1928, due to the important work of Piercy & Richardson (1928, 1930), Schuh (1953) and Kuethe (1958). Investigation of this phenomena was undertaken by Kestin & Wood (1970), Sutera *et al.* (1963) and Sutera (1965) at Brown University, in a separate way by Bearman (1972), and by Sadeh and his colleagues (1970, 1980). The Brown University group and Sadeh and his colleagues exposed experimentally the idea that short-wavelength disturbances (periodic along the span) are intimately related to and connected with perturbations within the boundary layer, while long-wavelength perturbations are not so related;

they delineated a critical wavelength and attempted a theoretical description of the phenomena.

Earlier, Görtler (1955) and Hämmerlin (1955) had devised a theory, related to that for Taylor and Görtler vortices, which showed that long-wavelength disturbances (periodic along the span), originating in the stagnation-point boundary layer, decay in the free stream. This work, associated with the theory of flow instability, was extended in significant ways by Wilson & Gladwell (1978) and by Brattkus & Davis (1991). The ideas presented in these papers are consistent with the experimental observations mentioned above, at least in relation to long-wavelength disturbances.

In the present work, we have considered disturbances which exist in the boundary layer but grow algebraically to much greater magnitudes outside the boundary layer. These disturbances have shorter wavelengths and play the role associated with such modes in the observations: they are intimately related to the interaction between the boundary layer and the external flow. Thus, our work extends the Görtler–Hämmerlin type of theory (with its significant later extensions) to the case of perturbations which do not decay at the edge of the boundary layer. Eigenmodes of both long and short wavelengths are now seen to fit into the picture enunciated by Kestin, Sadeh and their colleagues.

This work commenced in 1982. It is a pleasure to acknowledge the work of Dr M. E. Goldstein (1984), referred to above, for its insights into the problems of potential and viscous flow interactions.

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